Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #15

Integration Formulas – Guess and Check

Here are some integration formulas:

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log(ax+b) + C$$

$$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-b^2x^2}}\,dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right) + C$$

$$\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + C$$

Examples:

$$\int \frac{1}{5x+7}\,dx = \frac{1}{5}\log(5x+7) + C$$

$$\int \frac{\cos x}{\sin x}\,dx = \log \sin x + C$$

$$\int \frac{2x}{x^2+1}\,dx = \log(x^2+1) + C$$

Just Google "Table of Integrals"

Memorizing some formulas can speed up problem solving. It is a trade off. You have to pick your own optimum memorized list.

Guess and check

$$\int \frac{1}{1+(x-3)^2}\,dx$$

This looks like an inverse tan integral.

Take away the -3 and that's what you get.

Usually a -3 doesn't change the derivative much so lets guess that the anti-derivative is:

$$y = \tan^{-1}(x-3) + C$$

Check this with the chain rule:

$$\frac{dy}{dx} = \frac{1}{1+(x-3)^2}\frac{d(x-3)}{dx} = \frac{1}{1+(x-3)^2}$$

Guess, Check and Fix

If you guess and come close you may be able to fix your anti-derivative.

$$\int \frac{3x}{x^2+1}\,dx$$

Lets guess that the anti-derivative is close to

$$\log(x^2+1)$$

Then the derivative is:

$$(\log(x^2+1))' = \frac{1}{x^2+1}(2x) = 2\left(\frac{x}{x^2+1}\right)$$

We missed by a factor of $\frac{3}{2}$:

$$\int \frac{3x}{x^2+1}\,dx = \frac{3}{2}\log(x^2+1) + C$$